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Development of Turbulence Models for Free-Surface Flows

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14. ABSTRACT
The goal of this effort was to improve the ability to predict the free-surface flow in the near field of surface ships. The specific objectives were: to develop modeling approaches appropriate to the near-surface region in turbulent free surface flows and to obtain data for validation of CFD predictions of free-surface turbulent flows. The main results of this study can be summarized as follows: 1) The framework outlined in Hong & Walker (2000) was shown to be reasonable for turbulent free-surface flows. 2) The surface current is caused by a combination of turbulent surface fluctuations and turbulence anisotropy, with surface fluctuations dominating in high-Froude-number flows, and anisotropy otherwise. 3) The surface fluctuation effects can be approximated by a stress boundary condition. 4) A computational approach for predicting turbulence-generated waves was developed and the results compared favorably to experimental data. 5) Surface fluctuation measurements were carried out for a free-surface jet flow and the p.d.f. for the surface elevation was shown to be relatively Gaussian. 6) A method for measurement of the directional wave spectrum was developed and implemented and the directional spectrum of turbulence generated waves was characterized for a free-surface jet.

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1 Summary

This report summarizes work carried out under ONR Grant Number N00014-97-1-0053 at the University of Michigan, and coordinated efforts under ONR Contract Number N0014-00-C-0057 at Veridian Systems Division, Inc. The goal of this effort was to improve the ability to predict the free-surface flow in the near field of surface ships. The specific objectives were: to develop modeling approaches appropriate to the near-surface region in turbulent free surface flows and to obtain data for validation of CFD predictions of free-surface turbulent flows. The context of this modeling effort was the Reynolds-averaged Navier-Stokes (RANS) equations, as implemented in the code CFDSHIP-IOWA. The specific focus was the modeling of unsteady, unresolved surface waves generated by the near-field, turbulent flow around a surface ship, including the generation and propagation of these waves. The main results of this study can be summarized as follows: 1) The framework outlined in Hong & Walker (2000) was shown to be reasonable for the analysis and prediction of turbulent free-surface flows. 2) The surface current is caused by a combination of turbulent surface fluctuations and turbulence anisotropy, with surface fluctuations dominating in high-Froude-number flows, and anisotropy in low-Froude-number flows. 3) The surface fluctuations effects can be approximated by a modified stress boundary condition where the shear-stress is related to the gradient of the surface-fluctuation variance. 4) A computational approach for predicting the production and evolution of turbulence-generated waves was developed and the results compared favorably to experimental data. 5) Surface fluctuation measurements were carried out for a free-surface jet flow and probability density function for the surface elevation was shown to be relatively Gaussian. 6) A method for measurement of the directional wave spectrum was developed and implemented and the directional spectrum of turbulence generated waves was characterized for a free-surface jet.

2 Background

The results of Walker *et al.* (1995) for turbulent free-surface jets show that there is significantly more wave generation in high-Froude-number flows than in low-Froude-number flows. There is also a marked reduction in the turbulence kinetic energy levels in flows at high Froude number; the implication being that the turbulence kinetic energy is being transferred to wave energy, which then propagates away in the form of surface waves. This loss of energy (roughly 20% for $Fr = 8$, relative to $Fr = 1$) will affect the ultimate downstream evolution of the flow. Observations of the near-field flow around both full-scale and model-scale ships reveal significant turbulent wave generation in these flows. As a result, accurate prediction of these flows requires that the generation of waves by turbulence, as well as their subsequent propagation, be modeled appropriately. Since these waves are not resolved either spatially or temporally in Reynolds-averaged calculations, their effects also need to be included in the turbulence modeling for the sub-surface flow.

RANS codes such as CFDSHIP-IOWA (Tahara & Stern 1996) can model a fully deformable mean free surface, and they impose averaged versions of the kinematic and dynamic free-surface boundary conditions at the mean free surface. Careful examination of the instantaneous equations and boundary conditions, however, indicates that this approach is incompatible with unsteady surface fluctuations, and is fundamentally incorrect when such unresolved surface fluctuations are present. This is because the boundary conditions apply at the instantaneous position of the free surface, and averaging the equations does not allow the conditions to be applied at the mean free surface instead, except in the case where the mean and instantaneous free-surface locations are the same (i.e. no fluctuations).

Including the unresolved surface fluctuations in the Reynolds averaging process leads to a set of equations which applies across the entire air–water fluid volume (Hong & Walker 2000). One form for these equations is

$$\frac{\partial \bar{\rho}}{\partial t}+\frac{\partial}{\partial x_i}\bar{\rho}\langle U_i\rangle=0 \tag{1}$$

$$\begin{aligned}\bar{\rho}\left(\frac{\partial\langle U_i\rangle}{\partial t}+\langle U_j\rangle\frac{\partial\langle U_i\rangle}{\partial x_j}\right)=&-\frac{\partial\bar{P}}{\partial x_i}+\frac{\partial}{\partial x_j}\left[\bar{\mu}\left(\frac{\partial\langle U_i\rangle}{\partial x_j}+\frac{\partial\langle U_j\rangle}{\partial x_i}\right)\right]\\&+\frac{\partial}{\partial x_j}\left(\overline{\mu\left\langle\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right\rangle}\right)-\bar{\rho}\langle u_i u_j\rangle+\bar{\rho}g_i-\frac{\gamma}{\tilde{R}}f(z)\end{aligned} \tag{2}$$

where $\langle U_i\rangle=\overline{\rho U_i}/\bar{\rho}$ is a density-weighted (Favré) average, and the overbar indicates a time average; ρ, μ and γ are density, viscosity and surface tension, respectively, and $\tilde{R}$ is an average measure of the surface curvature. Here, $f(z)$ is the p.d.f. of the free-surface fluctuations, ρ_o and μ_o are the water density and viscosity, and, for a single-valued free surface,

$$\bar{\rho}(z)=\rho_o\left(1-\int_{-\infty}^{z}f(z)\,dz\right),\quad \bar{\mu}(z)=\mu_o\left(1-\int_{-\infty}^{z}f(z)\,dz\right) \tag{3}$$

at a given (x,y) position. The equation governing the mean free-surface position is

$$\rho_o\left(\frac{\partial G}{\partial t}+\langle U_i\rangle\frac{\partial G}{\partial x_i}\right)f(x_i)=\bar{\rho}\frac{\partial\langle U_i\rangle}{\partial x_i}, \tag{4}$$

which is derived from the time-averaged form of the continuity equation. Here $G=0$ defines the free surface. It should be noted that the set of equations (1), (2) and (4) reduce to the level-set method of Chang, *et al.* (1996) in the limit of laminar flow.

Solution of the above equations suffers from the same closure problems as the conventional RANS equations, with the additional unknown of the p.d.f. of the surface elevation fluctuations. In addition to the viscous terms associated with the mean strain rate, there is also an additional viscous term caused by the fluctuating strain-rate gradients; both of these are expected to be small in high-Reynolds-number free-surface flows. Assumption of a suitable functional form for $f(z)$ (e.g. Gaussian) reduces the closure problem to one of specifying the mean and variance of the surface elevation distribution (in addition to the specification of the Reynolds stresses). The mean surface elevation is known from (4), the surface elevation variance requires additional information.

If the surface-elevation fluctuations are driven primarily by the local sub-surface pressure field, then the local surface-elevation variance can be modeled in terms of the local turbulence properties. If, however, the main contribution to the surface fluctuations is in the form of waves, then the propagation and refraction of those waves must be modeled accurately in order to predict the surface fluctuations. The oceanographic community has, in recent years, developed statistical approaches to modeling the propagation of surface waves. These third-generation wave models (Komen *et al.* 1996) describe the evolution of the wave spectrum in space and time, and in so doing, retain the wave-like nature of the surface disturbances. They account for advection of wave energy by the fluid velocity as well as the wave group velocity (radiation effects), and interaction of the waves with velocity gradients (refraction, wave–current interactions). Since wave energy is not a 'scalar' quantity—each bit of energy has a propagation speed and direction associated with it—this type of wave spectrum model is the lowest-order statistical model which can be applied with the expectation of obtaining realistic results for turbulence-generated waves.

The action balance model is expressed as

$$\frac{\partial N}{\partial t}+\left(U_i+C_{gi}\right)\frac{\partial N}{\partial x_i}=k_i\frac{\partial U_i}{\partial x_i}\frac{\partial N}{\partial k_j}-\nu k^2N+S(x_i,k_i)\,; \tag{5}$$

here, $N = N(k_i,x_i)$is the wave action spectrum which varies in space and $N = E/\sigma(k)$, where E is the energy spectral density of the waves and σ is the intrinsic frequency of the waves for a given wavenumber, as determined from linear wave theory. C_{gi} is the vector group velocity of the waves, and S is a source term representing turbulent wave generation, or other effects. The second-from-last term in (5) represents viscous dissipation of wave energy, which impacts primarily the short waves.

3 Results

The objective of this study was to develop an approach to predicting turbulent free-surface flows in the context of the conventional Reynolds-averaged Navier–Stokes equations. In what is presented below, the main results are described. These include results on physical understanding in free-surface turbulent flows, model development, and single-point surface-elevation measurements, as well as wave spectrum measurements. These measurements can be used to guide development of, and serve to validate, modeling approaches for turbulence-generated waves.

The surface current, a large lateral which develops velocity near the free surface, is a major feature of many inhomogeneous turbulent free-surface flows (see Walker 1997 for a review). The new Reynolds averaged equations (2), above, forms the basis for a new analysis which parallels that of Walker (1997), but includes the effects of unsteady surface deformations. This more general analysis results in

$$\int\overline{\rho}\left[\hat{U}\frac{\partial\hat{V}}{\partial x}+\hat{V}\frac{\partial\hat{V}}{\partial y}+\hat{W}\frac{\partial\hat{V}}{\partial z}\right]dz\approx-\frac{\partial}{\partial y}\int\overline{\rho\left(v^2-w^2\right)}dz-\tfrac{1}{2}\rho_o g\frac{\partial\eta'^2}{\partial y}, \tag{8}$$

where η' is the r.m.s. surface fluctuation level. This indicates that the surface current is driven by the lateral gradient of either the Reynolds-stress anisotropy or the surface fluctuation variance. In high Froude number flows where the anisotropy is small, the surface fluctuation term dominates, while in low-Froude-number flows, the surface fluctuation variance is small, but the anisotropy is large. The details of the analysis leading to the above result, along a demonstration that the result is consistent with available experimental jet data appears in Hong & Walker (2000).

Surface fluctuation effects can only be included in non-level-set RANS codes in an approximate fashion. For small surface fluctuation levels, the surface fluctuation terms in (2) can be decoupled from the field equations and an effective dynamic free-surface boundary condition can be derived which includes the effects of surface fluctuations. The resulting approximate boundary condition is

$$\tau_{xz}=\mu\left(\frac{\partial\overline{U}}{\partial z}+\frac{\partial\overline{W}}{\partial x}\right)\approx-\tfrac{1}{2}\rho g\frac{\partial\eta'^2}{\partial x},\quad \tau_{yz}=\mu\left(\frac{\partial\overline{V}}{\partial z}+\frac{\partial\overline{W}}{\partial y}\right)\approx-\tfrac{1}{2}\rho g\frac{\partial\eta'^2}{\partial y}. \tag{9}$$

Here, the surface fluctuations are represented as an applied shear stress acting on the mean free surface. Details of the derivation are provided in Walker (2000).

A computational approach was developed in which the exact equations derived by Hong &Walker (2000) are reduced to an approximate form for small surface fluctuations. This approach was described in Walker (2000*b*, 2001*a*) and allows high-Froude-number flows with unsteady surface fluctuations to be treated with a conventional RANS code such as CFDSHIP-IOWA (Tahara & Stern 1996), rather than using a level-set approach. Surface fluctuations are modeled via coupled solution of a wave-action spectrum model their effect on the mean flow are captured using an approximate stress condition at the free surface. To accomplish this a wave-action balance solver was added to the CFDSHIP-IOWA code.

The effects of the free surface on the sub-surface turbulence at low Froude number are captured using a near-surface anisotropy model.

The approach developed were applied to turbulent free-surface jet flows, and comparisons of the results to experimental data were made in Walker (2001*a*, 2001*b*). First a low-Froude-number jet was calculated, where there was little wave generation and near-surface anisotropy dominates the behavior of the flow; here the approach performed quite well when compared to low-Froude-number jet data. A one-way-coupled prediction of a high-Froude-number jet was then presented, where the surface fluctuations were predicted using the wave-action model, but the surface fluctuations were not used in the boundary conditions for the subsurface flow (i.e. the subsurface flow is treated as a low-Froude-number flow). The predicted surface-fluctuation distribution was in good agreement with the observations. Finally, a two-way coupled calculation was presented, where the surface fluctuations were used in the boundary conditions on the subsurface flow. This agrees less well with the experiments; the magnitude of the outward velocity at the free surface was substantially under-estimated. This was believed to be due mainly to a lack of a complete model, capable of switching between the approximate stress condition in regions of large surface fluctuation, and an anisotropic near-surface turbulence model in regions with small surface fluctuations.

Surface-elevation measurements have been undertaken to quantify the fluctuation level for the free surface above a turbulent jet. These measurements were accomplished with a single-point laser-induced fluorescence technique where the water is dyed with a fluorescent dye, and the water surface is illuminated from above with an Argon–ion laser, and are described in Hong (2000). The fluorescent emission along the beam path is observed with a line-scan camera, and the bright-to-dark transition indicates the position of the free surface. These measurements were used to characterize the probability density function for the free-surface elevation as a function of position on the water surface. Comparison of plots of the experimentally determined p.d.f.s were made to Gaussian distributions calculated using the experimentally determined mean and standard deviation to establish that the free surface fluctuations are nearly Gaussian. At some locations, (e.g. on the centerline far downstream), the mode of the distribution is located slightly on the positive side of the mean. These results demonstrate and reinforce the fact that for this flow, and others of practical interest, the free surface position is not well defined and must be treated statistically.

The statistical description of the turbulence-generated wave field is the directional wave spectrum, indicating the energy density for waves of different wavenumbers (wavelengths) and propagation directions. The evolution of the wave spectrum is governed by a dynamical equation, in which the generation of waves by subsurface turbulence is represented by a source term, and the generated waves interact with mean velocity field, resulting in wave refraction and wave-energy changes locally. A method was developed which allows measurement of the directional wave spectrum. This approach has yielded the first-ever measurements of the wave spectrum of turbulence-generated waves.

To determine the wave spectrum, surface-elevation measurements over an area sufficient to determine the wavelength of the waves present are required. It is also necessary to determine the propagation direction of the waves. A snapshot of the wave field yields an estimate of wave direction, but there is a 180 degree directional ambiguity. To resolve this ambiguity (i.e. to determine whether the waves are moving to the left or right), temporal information is required.

Directional wave-energy spectrum measurements were accomplished using the method described in Lyzenga *et al.* (2003, in preparation). This approach uses a vertical laser sheet to illuminate the water surface along a line. The water contains a fluorescent dye, and the resulting fluorescent emission, which indicates the water surface elevation profile along the line of the laser sheet, is captured using a CCD camera. The laser is pulsed and pairs of images, separated by a short time interval ($\Delta t \approx 1\,\mathrm{ms}$), are

captured. A number of image pairs are captured and each image is then interrogated to determine the instantaneous one-dimensional surface elevation profile along the line defined by the laser sheet. From these profiles, the spatial cross-covariance function along the line for time lag Δt is estimated. The sheet is then rotated a few degrees about its vertical axis and the procedure repeated, building up a polar representation of the two-dimensional spatialcross-covariance function for time lag Δt. The directional wave spectrum for a given spatial location can be calculated from this two-dimensional cross-covariance function. Wave spectrum measurements were carried out for a free surface jet (Walker *et al.* 2002). The measured wave spectrum above the jet axis indicates that the dominant waves are propagating upstream against the jet velocity, this is consistent with the idea that energy would be concentrated in waves that are 'blocked' by the opposing velocity. Further from the jet axis, it is clear that the dominant waves are propagating outward, away from the jet axis, consistent with waves observed in the shadowgraph image.

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